GLOSSARY

absolute value. A number's distance from zero on the number line. The absolute value of –4 is 4; the absolute value of 4 is 4.

algorithm. An organized procedure for performing a given type of calculation or solving a given type of problem. An example is long division.

arithmetic sequence. A sequence of elements, a_1 , a_2 , a_3, . . . , such that the difference of successive terms is a constant, i.e., for every *i*, $a_i - a_{i-1} = k$; for example, the sequence {2, 5, 8, 11, 14, . . .} where the common difference is 3.

asymptote. An asymptote of a plane curve is a straight line such that the distance between a point on the curve and the line approaches zero as the distance between the point and the origin increases to infinity. For example, the x axis is the only asymptote of the curve sin(*x*)*/x.*

axiom. A statement about a mathematical system that is accepted without proof and from which theorems can be deduced. In a mathematical system that describes the points and lines in the plane, one example of an axiom would be the statement that there is a unique line through any two distinct points in the plane.

binomial. In algebra, an expression consisting of the sum or difference of two monomials (see the definition of *monomial*), such as 4*a* – 8*b.*

binomial coefficient. For *n* equal to any positive integer and for *k* equal to any integer between 0 and *n* (or 0 or *n* itself), the binomial coefficient B (*n, k*) is:

$$\frac{n!}{(n-k)!\ k!}$$

The most customary notations for B (*n, k*) are ${}_nC_k$ or $\binom{n}{k}$

The symbol $n!$ (n factorial) represents the product of all integers between 1 and n inclusive (e.g., $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$), except for 0! which is a special case and stands for 1 (i.e., $0! = 1$).

binomial distribution. In probability, a binomial distribution gives the probabilities of k outcomes A (or $n–k$ outcomes B) in n independent trials for a two-outcome experiment in which the possible outcomes are denoted A and B.

binomial theorem. The statement that, for positive integer n, $(a + b)^n$ is a polynomial having the binomial coefficient ${}_nC_k$ as the coefficient of the monomial term $a^k b^{n-k}$.

box-and-whisker plot. A graphical method for showing the median, quartiles, and extremes of data. A box plot shows where the data are spread out and where they are concentrated.

complex numbers. Numbers that have the form $a + bi$ where a and b are real numbers and i satisfies the equation $i^2 = -1$. Multiplication is defined by $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$, and addition is defined by $(a + bi) + (c + di) = (a + c) + (b + d)i$.

congruent. Two shapes in the plane or in space are congruent if there is a rigid motion that identifies one with the other (see the definition of *rigid motion*).

conjecture. An educated guess.

coordinate system. A rule of correspondence by which two or more quantities locate points unambiguously and which satisfies the further property that points unambiguously determine the quantities; for example, the usual Cartesian coordinates x, y in the plane.

corollary. A direct consequence of a theorem.

cosine. Cos(θ) is the *x*-coordinate of the point on the unit circle so that the ray connecting the point with the origin makes an angle of θ with the positive *x*-axis. When θ is an angle of a right triangle, then cos(θ) is the ratio of the adjacent side with the hypotenuse.

dilation. In geometry, a transformation *D* of the plane or space is a dilation at a point *P* if it takes *P* to itself, preserves angles, multiplies distances from *P* by a positive real number *r*, and takes every ray through *P* onto itself. In case *P* is the origin for a Cartesian coordinate system in the plane, then the dilation *D* maps the point (*x, y*) to the point (*rx, ry*).

dimensional analysis. A method of manipulating unit measures algebraically to determine the proper units for a quantity computed algebraically. For example, velocity has units of the form length over time (e.g., meters per second [*m/sec*]), and acceleration has units of velocity over time; so it follows that acceleration has units (*m/sec*)*/sec* = *m/*(*sec*^{2}).

expanded form. The expanded form of an algebraic expression is the *equivalent expression* without parentheses. For example, the expanded form of $(a + b)^2$ is $a^2 + 2ab + b^2$.

exponent. The power to which a number or variable is raised.

exponential function. A function commonly used to study growth and decay. It has the form $y = a^x$ with *a* positive.

factors. Any of two or more quantities that are multiplied together. In the expression 3.712×11.315, the factors are 3.712 and 11.315.

field. A “number system” which resembles the system of rational numbers in that members can be multiplied and added, and there is a zero and a unit (named *one*), and the rules governing the arithmetic combinations are familiar ones. For

example, for any *a, b, c:* $ab = ba;$ $1 \cdot a = a;$ $0 + a = a;$ $a + b = b + a;$ $a(b + c) = a \cdot b + a \cdot c;$ and the equations $a \cdot x = b$ (unless $a = 0$) and $a + x = b$ always have unique solutions. The complex numbers, the real numbers, and the rational numbers all form fields. There are other fields (e.g., all real numbers of the form $a + b\sqrt{3}$).

function. A correspondence in which to each value of one variable there corresponds only one value of another..

geometric sequence. A sequence in which there is a common ratio between successive terms. Each successive term of a geometric sequence is found by multiplying the preceding term by the common ratio. For example, in the sequence {1, 3, 9, 27, 81, . . .} the common ratio is 3.

heuristic argument. The term universally used in mathematics for an argument that is suggestive of the truth of a mathematical statement but which is not entirely logically correct or complete.

histogram. A vertical block graph with no spaces between the blocks. It is used to represent frequency data in statistics.

hypothesis. Synonymous with *assumption.*

inequality. A relationship between two quantities indicating that one is strictly *less than* or *less than or equal* to the other.

integers. The set consisting of the positive and negative whole numbers and zero; for example, {. . . -2, -1, 0, 1, 2 . . .}.

irrational number. A real number that cannot be represented as an exact ratio of two integers, such as π or the square root of 2.

lemma. A true statement of lesser significance than a theorem, usually isolated as an interim statement in the course of a longer chain of reasoning.

linear equation. An equation stating that a linear expression equals zero.

linear expression. An expression of the form $ax + b$ where x is variable and a and b are constants; or in more variables, an expression of the form $ax + by + c$, $ax + by + cz + d$, etc.

logarithm. A logarithm is an inverse of an exponential. The equation $y = a^x$ can be written as $x = \log_a y$, meaning x is the logarithm of y to the base a. Any positive number except 1 can be used as the base for a logarithm function (logarithms to base 10 are called *common logarithms* and logarithms to base e are called *natural logarithms*).

mean. In statistics, the average obtained by dividing the sum of two or more quantities by the number of these quantities.

median. The middle point in an ordered set of data. If N (the number of points in the set) is odd, the median is the single value in the middle, namely the value with rank (N + 1) / 2. If N is even, there is not a single value in the middle, so the median is defined to be the mean of the two middle values, namely the values with ranks N/2 and N/2 + 1.

mode. In statistics, the value that occurs most frequently in a given series of numbers.

monomial. In the variables x, y, and z, a monomial is an expression of the form $ax^m y^n z^k$, in which m, n, and k are nonnegative integers and a is a constant (e.g., $5x^2$, $3x^2y$ or $7x^3yz^2$).

nonstandard unit. Unit of measurement expressed in terms of objects (such as paper clips, sticks of gum, shoes, etc.).

parallel. In Euclidean geometry, two distinct lines are said to be parallel if they have no points of intersection. Two distinct lines in the coordinate plane are parallel if and only if they have the same slope.

permutation. A permutation of the set of numbers {1, 2, . . . , *n*} is a reordering of these numbers.

polar coordinates. The coordinate system for the plane is based on *r* (the distance from the origin) and θ (the angle between the positive *x*-axis and the ray from the origin to the point).

polar equation. Any relation between the polar coordinates (r, θ) of a set of points (e.g., $r = 2\cos\theta$ is the polar equation of a circle).

polynomial. In algebra, a sum of one or more monomials; for example, $-3.5x$, or $x^2 + 2xy + y^2$.

postulate. Synonymous with *axiom.*

prime. A natural number *p* greater than 1 is prime if and only if the only positive integer factors of *p* are 1 and *p*. The first seven primes are 2, 3, 5, 7, 11, 13, 17.

probability space. A set of entities called events, to each of which is assigned a number called its probability. For example, when one throws a pair of dice five times, then an event might be *obtaining the result 12 every time.* The associated probability for this example event is $(1/36)^5$.

quadratic function. A function *f* is called a quadratic function if it can be written in the form $f(x) = ax^2 + bx + c$, where *a, b,* and *c* are real numbers and $a \neq 0$. Note that a quadratic function is a polynomial of degree 2.

quartiles. The term *quartiles* sometimes refers to quarters of a rank-ordered set of data, but it more commonly refers to the three cut points or boundaries that divide

an ordered data set into four groups with an equal number of elements in each group. The second quartile cut point is defined as the median. There are minor variations in the formal definitions for the lower (or first) and upper (or third) quartile cut points that can yield different answers depending on the number of elements in the set. One defines them as the medians of the data points below and above the median, excluding the median in the case of an odd number of points. Another includes the median in both cases.

This Framework adopts a definition of the quartile cut points that can be expressed rather formally as:

The inverse of the empirical cumulative distribution function with means taken at the points of discontinuity, evaluated at 0.25, 0.50, and 0.75 for the lower or first quartile, median or second quartile, and upper or third quartile, respectively.

One way to represent this is with a line segment from 0 to 1, divided into *N* equal parts (*N* is the number of elements in the set) for the elements in rank order. The length of each small segment is 1/*N*. The lower quartile is the element whose small segment includes the point 0.25, and the upper quartile is the element whose small segment includes the point 0.75. When the data set consists of an integral multiple of 4 points, the mean of the two points adjacent to the cut point is taken as the quartile cut point. This definition is consistent with the method used to find the *median* for sets with an even number of elements.

Let *N* be the number of points in the set and *Int*() mean convert to an integer by truncation.

When the number of elements is not divisible by four, the lower and upper quartile cut points are the values of the elements with rank *Int*(*N*/4)+1 and *Int*(3*N*/4)+1.

When the number of elements is divisible by four, the lower quartile cut point is the mean of the values of the elements whose ranks are N/4 and (N/4)+1, and the upper quartile cut point is the mean of the values of the elements whose ranks are 3N/4 and (3N/4)+1.

random variable. A function assigning a number to each event in a probability space.

range. In statistics, the difference between the greatest and smallest values in a data set. In mathematics, the image of a function.

ratio. A comparison of two numbers, often expressed by a fraction. For example, if there are three boys in class for every two girls, the ratio of boys to girls is 3:2 or 3/2 (read as 3 to 2).

rational numbers. Numbers that can be expressed as the quotient of two integers; for example, 7/3, 5/11, –5/13, $7 = 7/1$.

real numbers. The set of all decimal expressions, finite or infinite in length.

reflection. The reflection through a line in the plane or a plane in space is the transformation that takes each point in the plane to its mirror image with respect to the line or its mirror image with respect to the plane in space. It produces a mirror image of a geometric figure.

rigid motion. A transformation of the plane or space, which preserves distance and angles.

root extraction. Finding a number that can be used as a factor a given number of times to produce the original number; for example, the fifth root of $32 = 2$ because $2 \times 2 \times 2 \times 2 \times 2 = 32$.

rotation. A rotation in the plane through an angle θ and about a point *P* is a rigid motion *T* fixing *P* so that if *Q* is distinct from *P,* then the angle between the lines

PQ and *PT(Q)* is always θ. A rotation through an angle θ in space is a rigid motion *T* fixing the points of a line *l* so that it is a rotation through θ in the plane perpendicular to *l* through some point on *l*.

scalar matrix. A matrix whose diagonal elements are all equal while the nondiagonal elements are all 0. The identity matrix is an example.

scatterplot. A graph of the points representing a collection of data.

scientific notation. A shorthand way of writing very large or very small numbers. A number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10 (e.g., $7{,}000 = 7 \times 10^3$ or $0.0000019 = 1.9 \times 10^{-6}$).

sieve of Eratosthenes. A method of getting all the primes in a certain range, say from 2 to 300. Start with 2, cross out all numbers from 2 to 300 which are multiples of 2 but not equal to 2. Go to the next remaining number, which is 3. Now cross out all numbers up to 300 which are multiples of 3 but not equal to 3. Go to the next remaining number, which is 5. Cross out all remaining numbers which are multiples of 5 but not equal to 5. And so on. At each stage, the next number is always a prime. At the end of this process, when there are no more numbers below 300 to be crossed out, every remaining number is a prime. (For the case at hand, once multiples of 17 other than 17 itself have been crossed out, the process comes to an end since the product of any two primes greater than 17 must be greater than 300.)

similarity. In geometry, two shapes *R* and *S* are similar if there is a dilation *D* (see the definition of *dilation*) that takes *S* to a shape congruent to *R*. It follows that *R* and *S* are similar if they are congruent after one of them is expanded or shrunk.

sine. Sin (θ) is the *y*-coordinate of the point on the unit circle so that the ray connecting the point with the origin makes an angle of θ with the positive *x*-axis. When θ is an angle of a right triangle, then *sin* (θ) is the ratio of the opposite side with the hypotenuse.

square root. The square roots of *n* are all the numbers *m* so that $m^2 = n$. The square roots of 16 are 4 and –4. The square roots of –16 are 4 *i* and –4 *i.*

standard deviation: The *standard deviation* is a measure of dispersion or variability among the points in a set of data. It can be interpreted as the average (i.e., typical, not literally the mean) deviation (distance) of a point from the mean of the distribution. More precisely, it is the square-root of the average of the squared deviations of the points from the mean of the distribution (the phrase *root of the mean square* is encountered in some disciplines).

The standard deviation is also the square-root of the variance. Just as there are two formulations for variance (the *population variance* and the *sample variance*), there are two formulations for the standard deviation. The *population standard deviation* is the square-root of the population variance. The *sample standard deviation* is the square-root of the sample variance.

symmetry. A symmetry of a shape *S* in the plane or space is a rigid motion *T* that takes *S* onto itself *(T(S) = S).* For example, reflection through a diagonal and a rotation through a right angle about the center are both symmetries of the square.

system of linear equations. Set of equations of the first degree (e.g., $x + y = 7$ and $x - y = 1$). A solution is a set of numbers that, when it replaces variables, renders the equations true. For the present example, "$x = 4$ and $y = 3$" is a solution.

theorem. A significant true statement in mathematics, which is ultimately of the form "*p* implies *q*," where *p* represents a set of hypotheses and *q*, a conclusion.

translation. A rigid motion of the special form $x \to x + v$ for all x in the plane or in space, where v is a fixed vector defining the motion.

transversal. In geometry, given two or more lines in the plane a transversal is a line distinct from the original lines and intersecting each of the given lines at single point.

unit fraction. A fraction of the form 1/*n*, where *n* is a positive integer.

variable. A placeholder in algebraic expressions; for example, in $3x + y = 23$, x and y are variables.

variance: Variance is a measure of dispersion or variability among the points in a set of data. It can be interpreted as the average squared deviation (distance) of the points from the mean of the distribution. For a population (that is, when we have all of the data points for whatever group is being evaluated), this is commonly written as

$$\sum_{i=1}^{N} (X_i - \overline{X})^2 / N$$

Where:

X	is the set of elements
X_i	is the *i*th element in the set of elements
$\overline{X}$	is the mean of the entire set
N	is the number of elements in the set, and
$\sum_{i=1}^{N}$	indicates summation for elements 1 to N

This formulation is often called the *population variance* and is often represented as σ^2.

When we are dealing with a sample (that is, a subset of the complete population), we cannot of course compute the mean and variance of the population exactly, so

we estimate them. When applied to a sample, the formula above tends to underestimate the true variance. An unbiased estimate is computed as:

$$\sum_{i=1}^{N}(X_i - \overline{X})^2 / (\mathrm{N} - 1)$$

This formulation is typically called the *sample variance*, and is often represented as s^2.

vector. In physics, a measurable quantity such as force, which has both a magnitude and a direction, and sometimes also a point of application. In mathematics, a vector is a member of an algebraic system that has addition among its members and multiplication by real numbers (called scalars), with the entire system obeying certain algebraic rules resembling the manner in which the vectors of physics may be combined.

zeros of a function. The points at which the value of a function is zero.